Necessity and Truth Makers

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I met Barry for the first time at the Ingarden Symposium held in Cracow in 1984. We both co-chaired the session in formal ontology. At the beginning, Barry said that formal ontology must be done in military order. He meant that everything must be done on time. However, I prefer to replace the term 'military' with the term 'logical' because I believe that formal ontology must be done according to logical order. It is not only possible but, I dare say, it is quite certain that Barry's understanding of logic is different than mine. In light of this difference, I recognize that I have a challenging task ahead since I have chosen to address 'truth makers' here as my celebratory contribution for Barry's Festschrift. I hope that he will be tolerant with respect to my formalist attitude, at least to some respect.

I rather follow a Polish imperative to understand logic in a restrictive sense, that is, formal in the traditional sense. Thus, I should add that my analysis in this paper follows the Polish analytic tradition. I know very well that Barry has always shown great respect for Polish analytic philosophy as a sound mode of philosophizing. But I also know that he would object to my identifying the philosophy in Poland as Polish analytic philosophy because, to him, good philosophy is not characterized by national idiosyncrasies. He used to say, "There is no such thing as Polish philosophy; it is just good philosophy." His point was that to qualify analytic philosophy with the term 'Polish' renders the phrase ambiguous: it could refer to a particular national philosophy or to good philosophy done in Poland. If the latter, then there is no need to say anything other than 'good philosophy.'

In genuine observance of logical order and good philosophical fashion, then, I shall start my examination with some clarifications. First of all, I do not belong to the advocates of the theory of truth-makers. More precisely, I favor the semantic theory of truth (see Woleński 2014) and Tarski's truth-definition over defining the concept of truth via truth-makers. Eventually, truth-making may be used in explaining the criteria of truth, but I shall skip this here. Nonetheless, I will not argue against 'truth-makerism' as a general account of how the concept of truth should be defined. In other words, I am interested in internal problems of Truth-Maker Theory (hereinafter TMT). My only task here consists in analyzing the consequences of seeing truth-makers via the idea that they necessitate truth.

The view that truth-makers necessitate truth of sentences—e.g., statements, propositions, judgments, etc.—has many advocates in contemporary discussions (see Smith 1999; Armstrong 2004; Merricks 2007).[1] On the other hand, the idea of truth-making as a necessary nexus does not occur in the seminal paper (see Mulligan, Simons, and Smith 1984), which opened the present debate about truth-makers. A fundamental intuition in regard to the truth-making relation is captured by

(1) A truth-bearer is true if and only if it has a truth-maker.

In a formal language (see Rami 2009, p. 3), (1) as the truth-maker principle can by expressed by

(2) For every A, A is true if and only if there is a y such that y is a truth maker for A.
In symbols: $\forall A(\mathbf{Tr}(A) \Leftrightarrow \exists y\mathbf{TrMk}^{\mathbf{A}}(y))$.

Yet I consider (2) as less convenient for analysis than its conversion into something similar to the T-scheme, namely[2]

(3) Every instance of the scheme
(*) $\mathbf{Tr}(A) \Leftrightarrow \exists y\mathbf{TrMk}^{\mathbf{A}}(y)$ is a theorem of **TMT** (truth-makers theory)

Now, the problem here is whether the necessity parameter must be introduced into (*). The truth-maker necessarists—i.e., those who believe that truth-making is just necessary—reply "Yes" and propose various ways in order to justify this strong claim. I will examine this claim by using modal logic.

I would like to recognize at this juncture that, in the foregoing, I have entirely neglected to address the nature of truth-makers as well as the problem of whether (2) suffices for developing a full-blooded theory of truth-makers. I only assume that they are things of a sort without deciding whether they are mereological or set-theoretical entities, and without considering additional constraints such as the principle of projection proposed by Barry Smith (see Smith 1999). Furthermore, I assume that modal logic—in fact, very elementary principles of modality—can be applied to **TMT** independently of the ontological status of truth-makers. This assumption is commonly shared by many truth-maker theorists, including Barry Smith, who says (see Smith 1999, p. 277) that he uses modal logic "in the vicinity of S4." All propositional functors, that is, negation, implication, etc., have the classical truth-functional interpretation.[3]

The simplest way to cope with the problem is to split (3) into

(4) (a) $\mathbf{Tr}(A) \Rightarrow \exists y\mathbf{TrMk}^{\mathbf{A}}(y)$.
(b) $\exists y\mathbf{TrMk}^{\mathbf{A}}(y) \Rightarrow \mathbf{Tr}(A)$.

Now (4a) states that $\mathbf{Tr}(A)$ is the sufficient condition for $\exists y\mathbf{TrMk}^{\mathbf{A}}(y)$)—i.e., being true is sufficient for the existence of a truth-maker—but (4b) considers $\exists y\mathbf{TrMk}^{\mathbf{A}}(y)$ as the necessary condition for $\mathbf{Tr}(A)$. One could even say that (4b) nicely captures the significant sense in which truth-making just necessitates truth itself. However, we can easily see that this account appears as artificial and too poor for necessarianism.

Another possibility is to change the succession in (3) in order to obtain its equivalent, that is

(5) $\exists y\mathbf{TrMk}^{\mathbf{A}}(y) \Leftrightarrow \mathbf{Tr}(A)$.

Under this move, $\mathbf{Tr}(A)$ is necessary for $\exists y\mathbf{TrMk}^{\mathbf{A}}(y)$, but the latter is sufficient for the former. Clearly, (4) and (5) make the entire problem trivial because both $\exists y\mathbf{TrMk}^{\mathbf{A}}(y)$ and $\mathbf{Tr}(A)$ mutually co-necessitate and co-suffice. It is not surprising that this situation is caused by (3). This constraint claims

(6) $\mathbf{TMT} \vdash$ (or $\vdash^{\mathbf{TMT}}$) $\mathbf{Tr}(A) \Leftrightarrow \exists y\mathbf{TrMk}^{\mathbf{A}}(y)$, for any A.

This means that instances of (*) are theorems. Assuming that we have sufficient deductive resources, for instance the ω-rule, we can even say that the formula "$A(\mathbf{Tr}(A) \Leftrightarrow \exists y\mathbf{TrMk}^{\mathbf{A}}(y))$ is provable in **TMT**.

We can draw some lessons come from the foregoing elementary observations. The first is that necessary and sufficient conditions can be reversed. The explanation is as follows. If a given theory **Th** proves the formula $A \Leftrightarrow B$ (in symbols, $\mathbf{Th} \vdash (A \Leftrightarrow B)$), the way of structuring necessary and sufficient conditions is conventional to some extent. Basically, proving the formula of the type $A \Leftrightarrow B$ requires to demonstrate two implications, namely $A \Rightarrow B$ (the sufficient condition) and $B \Rightarrow A$ (the necessary condition).

Secondly, one might eventually say that (*) is **TMT**-necessary or **TMT**-analytic. Staying with necessity as more relevant in the present context than is analyticity, we should examine which kind of necessity is involved in the discussed issue. Clearly, (*) is not a tautology, similarly as **T**-scheme (see Woleński 2008). Hence, if we agree to speak about **TMT**-necessity, it is of a conditional character. In other words, **TMT**-necessity (and eventually, other modalities) are not logical. I will return to this below.

Thirdly, since according to the principles of propositional calculus, we can decompose (*) into the disjunctive formula

(**) $\mathbf{Tr}(A) \wedge \exists y\mathbf{TrMk}^{\mathbf{A}}(y) \neg\, \mathbf{Tr}(A) \wedge \exists y\mathbf{TrMk}^{\mathbf{A}}(y)$,

the following question arises: whether or not 'being not-true' and 'being false' are actually equivalent predicates.

The above considerations suggest that a weakening of (*) to (4b) actually helps in analyzing at least some aspects of introducing necessity into the logico-philosophical business of truth-making. Thus, we should consider how embed the sign of necessity into the formula $\exists y\mathbf{TrMk}^{\mathbf{A}}(y) \Rightarrow \mathbf{Tr}(A)$. I begin with a proposal of Trenton Merricks.[4] He writes:

> *Necessitarianism* says that a truthmaker necessitates that which it makes true. That is, necesitarianism says that, for all x and all p, x is a truthmaker for p only if x's mere existence is metaphysically sufficient for p's truth.

Let me try to formalize this definition. It can be done, by the formula (and I slightly change letters according to my notational conventions; the symbol **Nec** stands for 'necessitates').

(7) $\forall y \forall A(\mathbf{Ex}(\mathrm{y}) \wedge \mathrm{y} \in \mathbf{TrMk^A}(y)\ \mathbf{Nec}\ \mathbf{Tr}(A))$.

I take the word 'only' as indicating that the formula ⇒should precede the expression $\mathbf{Tr}(A)$, I render 'mere existence' by 'y exists' (I skip the problem of the status of this expression; one can consider it as a special predicate), and 'metaphysically sufficient' by 'y belongs to truth-makers of A' without entering into various questions concerning metaphysical grounding or dependence.

Merricks (2007) adds:

> Understood as a necessary condition for *making true*, necessitarianism is now truthmaker orthodoxy. […]. Let *conditional necessitarianism* be the denial of necessarianism conjoined with the claim that for all x and p, if x is a truthmaker for p*,* then, necessarily, if both x and p exist, then p is true. […]. Conditional necessarianism is equivalent to the claim that if x is truthmaker for p, then it is impossible that x exists and p have a truth-value other than true (or lacks truth-value altogether).[5]

The first sentence of the last quotation has to be corrected. Independently of what is necessarianism and how this view can or should be understood, it cannot be identified with functioning as a necessary condition that truth-makers produce truth of truth-bearers. It is clear that necessarianism proposes how to define the necessary constraint for the relation of making true, for instance, by (7) saying that the existence of a truth-maker y necessitates that a proposition A is true.

How to formalize the conditional necessarianism? First, it has the denial of (7) as its component. Thus, we have the following sequence of formulas:

(8) (a) $\neg(\forall y \forall A(\mathbf{Ex}(\mathrm{y}) \wedge y \in \mathbf{TrMk^A}(y)\ \mathbf{Nec}\ \mathbf{Tr}(A))$;
(b) $\neg \exists y \neg \forall A(\mathbf{Ex}(\mathrm{y}) \wedge y \in \mathbf{TrMk^A}(y)\ \mathbf{Nec}\ \mathbf{Tr}(A))$;
(c) $\neg \exists y \neg \exists A \neg(\mathbf{Ex}(\mathrm{y}) \wedge y \in \mathbf{TrMk^A}(y)\ \mathbf{Nec}\ \mathbf{Tr}(A))$;
(d) $\neg(\mathbf{Ex}(\mathrm{y}) \wedge y \in \mathbf{TrMk^A}(y)\ \mathbf{Nec}\ \mathbf{Tr}(A))$???

The steps from (8)(a) to (8)(c) are justified by simple rules for negating quantifiers in the classical first-order logic. Why does question marks occurs in the assertion (8)(d)? The problem refers to the place in which the sign of negation should occur as related to **Nec** (for simplicity, I dropped quantifiers in (8)(d)). For convenience, let the box □ stand for **Nec**. The first possibility is to convert the problematic (8d) to

(9) $\mathbf{Ex}(\mathrm{y}) \wedge y \in \mathbf{TrMk^A}(y) \wedge \neg \Box \mathbf{Tr}(A)$,

saying that ¬ (7) means three things: (a) that an item exists; (b) it belongs to truth-makers of A, and (c) it is still not necessary that A is true. The second possibility consists in adopting the formula

(10) $(\mathbf{Ex}(\mathrm{y}) \wedge y \in \mathbf{TrMk^A}(y) \wedge \Box \neg \mathbf{Tr}(A)$,

as the denial of (7). However, one could observe that

(11) $\neg \Box(\mathbf{Ex}(\mathrm{y}) \wedge y \in \mathbf{TrMk^A}(y) \wedge \mathbf{Tr}(A))$,

(12) $\Box \neg((\mathbf{Ex}(\mathrm{y}) \wedge y \in \mathbf{TrMk^A}(y) \wedge \mathbf{Tr}(A))$,

better fit logical intuitions of denying (7), but (12) appears as much more coherent with the view of necessarianism for maintaining that it is impossible that truth-makers for A exist, but it is not true. Yet (see comments on (**) above) the problem remains whether not-true means false, possessing another logical value than being true-or-false or indicates a truth-value gap. Thus, the choice of basic logic can be fairly relevant. In order to simplify the issue, I interpret 'A is not true' as 'A is false', but nothing depends on this setting in my further considerations.

The positive content of conditional necessarianism has its rendering in the formula (proposed by Merricks in the above quoted passage)

(13) $\forall A \forall y(y \in \mathbf{TrMk^A}(y) \Rightarrow \Box((\mathbf{Ex}(A) \wedge \mathbf{Ex}(y) \Rightarrow \mathbf{Tr}(A))$.

Accordingly, the conditional necessarianism is the conjunction of (12) ∧ (10); (12) for the use of impossibility by Merricks himself. However, we should check whether this conjunction is actually equivalent to

(14) $y \in \mathbf{TrMk^A}(y) \Rightarrow \Box(\mathbf{Ex}(y) \wedge \mathbf{Tr}(A))$.

Merricks is not right, because (14) as a consequence of (10) cannot be equivalent with its antecedent conjoined with something else, for instance, (13) in the considered case. (14) suggests still different interpretation of necessarianism to be obtained by weakening (7) understood as a conjunction $\mathbf{Ex}(y) \wedge \boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}}(y) \wedge \Box(\mathbf{Tr}(A)$ to the implication

(15) $\mathbf{Ex}(y) \wedge \boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}}(y) \Rightarrow \Box(\mathbf{Tr}(A)$

The weakening in question consists in the fact that (15) is a consequence of (7) in the adopted interpretation.

Yet (15) does not close the issue. Having that necessarianism (its conditional version is a slight variant of what Merrricks simply calls necessarianism) has its proper rendering in the implication $\mathbf{Ex}(y) \wedge \boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}}(y) \Rightarrow (\mathbf{Tr}(A)$ with the added necessity component, we should decide in which place the operator $\Box$**.** We have the following possibilities (for simplicity and work with schemes similar to (*)):

(16) (a) $\Box(y \in \mathbf{TrMk}^{\mathbf{A}} \Rightarrow \mathbf{Tr}(A))$
(b) $\Box(y \in \mathbf{TrMk}^{\mathbf{A}}) \Rightarrow \Box\mathbf{Tr}(A))$
(c) $\Box(y \in \mathbf{TrMk}^{\mathbf{A}}) \Rightarrow (\mathbf{Tr}(A))$
(d) $y \in \mathbf{TrMk}^{\mathbf{A}} \Rightarrow \Box\mathbf{Tr}(A)$
(e) $y \in \mathbf{TrMk}^{\mathbf{A}} \Rightarrow^{\Box} (\mathbf{Tr}(A)$

Since (16)(b) follows from (16)(a), it is redundant. (16)(c) is implausible, because its antecedent is stronger than its consequent. Using the intuitive possible world semantics, it can happen that A is false at some world which verifies the sentence $\boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}}$. Due to the modal principle $\Box A \Rightarrow A$, necessity of $\mathbf{Tr}(A)$ is reducible to the factuality of $\boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}}$, unless we adopt a very strong modal logic in which every truth is necessary. This logic requires the Gödel rule $A \vdash \Box A$ without restriction that A is a tautology. This solution, although possible for necessarianism, obscures the difference between truths of logic and factual truths.

In (16)(e), necessity qualifies the relation between the antecedent and the consequent. If we read this formula as '$y \in \mathbf{TrMk}^{\mathbf{A}}$ entails $\mathbf{Tr}(A)$', we have that $\boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}} \vdash \mathbf{Tr}(A)$ and, by using the deduction theorem, we obtain $\vdash(\boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}} \Rightarrow \mathbf{Tr}(A))$. Consequently, the formula $\boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}} \Rightarrow \mathbf{Tr}(A)$ belongs to theorems. Of course, we should precede $\vdash$ by **TMT**, because we assume that we work in the framework of the theory of truth-makers. Nothing prevents identification of $\vdash^{\mathbf{TMT}}$ with $\Box^{\mathbf{TMT}}$, that is **TMT**-entailment with **TMT**-necessity. This suggests that (16)(a) can (should?) we written as $\Box(\boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}} \Rightarrow \mathbf{Tr}(A))$.

(17) $\Box^{\mathbf{TMT}}(\boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}} \Rightarrow \mathbf{Tr}(A))$.

The above reasoning shows that 16(e) is equivalent to (16)(a), provided that **TMT**-entailment and **TMT**-necessity are co-extensional.

Logic is a good example for showing that (17) works. If the box $\Box$ expresses logical necessity, we can drop the upper index. So we have $\Box(\boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}} \Rightarrow \mathbf{Tr}(A))$ and, by modal logic, $\Box(\boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}}) \Rightarrow \Box\mathbf{Tr}(A))$. The formula $\Box(\boldsymbol{y} \in \mathbf{TrMk}^{\mathbf{A}})$ says that every possible world contains a truth-maker for A, which is necessary true that is, y is true in every possible world. So A is a truth of logic. I cannot recommend an automatic repetition of this reasoning for the conditional necessity instantiated by $\Box^{\mathbf{TMT}}$. Although I assumed that it is co-extensional with $\vdash^{\mathbf{TMT}}$ this assertion requires further comments. In particular, although the symbol $\vdash$ has the standard meaning in the context of **TMT**-entailment, it is problematic whether logical necessity and conditional necessity are species of a general necessity concept. The sign $\Box^{\mathbf{TMT}}$ expresses *de dicto* modality.

On the other hand, the necessiarists about truth-makers want to speak about necessitation *de re*. In other words, (17) proposes an extensional approach to necessity, but (7) invites an intensional theory. Hence, (17) is probably too weak for developing the view that truth-makers necessitate truth of truth-bearers and make this job *de re.* Anyway, in my opinion, (17) says everything that can be said on the necessity of the truth-making relation on the basis of logic. In fact, the above analysis allows the conclusion that truth-making necessitates truth in the sense of being its necessary condition. Saying more precisely, the true assertion that y is a truth-maker for A acts as a necessary condition for the correct assertion that A is true. If someone considers a further metaphysical or/and ontological analysis of truth-making as required, he or has to go beyond logic.[6] The question is, do we need to go beyond logic with regard to propositions about empirically recognized facts? Since I am writing this paper to celebrate my friend Barry, consider the sentence

(18) Barry and Jan are friends.

What is the truth-maker of (18)? Let us say that the instance of the relation of friendship is **RF**. In other words, this relation obtains in the case of Barry and Jan, that is, <Barry, Jan> $\in$ **RF**. What does it mean to say that the truth-maker in question necessitates (18)? The first possibility is that there is a real connection (metaphysical, ontological, causal, etc.), which necessitates the sentences saying that Barry and Jan are friends. I must confess that I am not able to translate this nexus into exact semantic terms. Certainly, it is not necessary that <Barry, Jan> $\in$ **RF**. Although it occurs in the actual world, but it could be otherwise to my great regret. The second possibility is that (18) is a necessary truth. However, it is not, because it is not true in all possible worlds, but, fortunately, it holds in the actual world. This exactly means that (18) and <Barry, Jan> $\in$ **RF** are connected in a particular world and perhaps in some conceptual replicas of it. In modal semantics of possible worlds, what is so-called the real world is distinguished as the point of reference for the accessibility relation, and the status of this object is precisely the same as any other world. Simply speaking, it is a model (or algebraic structure). Any serious talk about truth-makers requires an assumption that what is actual, has the privileged metaphysical character as something really existing and remaining in some ontological relations, like necessity or possibility. Yet the deep difference between ontology and formal semantics for modalities must be taken into account, otherwise it essentially obscures the entire issue. Since my approach uses the second, it is sufficient for me to say that, if this mode of speaking is preferred, truth-bearers of sentences have logical values in the real world according to truth-makers. Involving necessity does not contribute to the discussed issue, unless someone explains the concept of metaphysical necessitation.

My view is that logical necessity is the only well-defined kind of the concept expressed by the symbol $\Box$ and its meaning is captured by truth in all models, that is, validity. We can eventually introduce a more general notion, namely being valid in a specified class of models, which is determined by a set of axioms. However, necessity as related to a single model, seems to be an oddity. This is the reason to skip necessitation as attributed to truth-making. Perhaps my friend Barry would agree, or perhaps he would see things otherwise. Whatever the case, he will remain my true friend regardless.

NOTES

1 I prefer the phrase 'truth-maker' over 'truthmaker' (similarly in the case "truth-making"; this convention does not concern quotations employed in this paper.

2 I omit restrictions required for the Liar paradox.

3 See Restall (1996) for an analysis based on relevant conditionals.

4 See Merricks (2007), Chapters 1–2, p. 5.

5 Merricks, 2007, pp. 5, 7.

6 See papers in Beebee and Dodd (2005), Monmoyer (2007), and Lowe and Rami (2009) for the present state of the debate on truth-makers.

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